Beam Dynamics Issues in Linear non-Scaling FFAGs

J. Scott Berg
Brookhaven National Laboratory
NuFact 06 WG3
28 August 2006

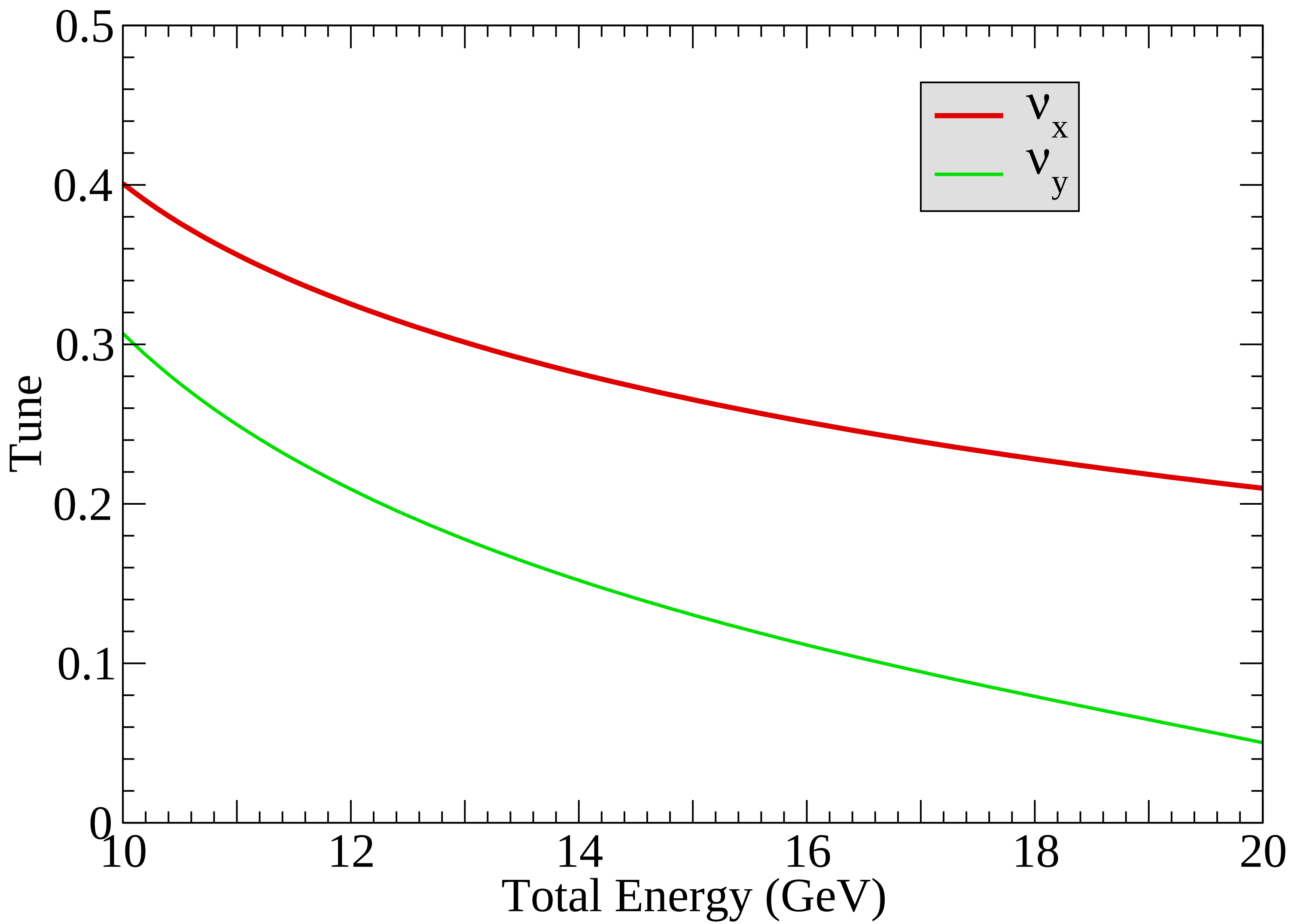
Outline

- Error tolerance
- Longitudinal dynamics
- Time of flight dependence on transverse amplitude
- Electron model: EMMA

Basic Design Principles

- Tune depends on energy: pass through resonances
- Use linear magnets to avoid driving nonlinear resonances
- Maintain symmetry (short, identical cells) to avoid driving linear resonances
    - Errors break this symmetry
- Accelerate rapidly through remaining weakly driven resonances

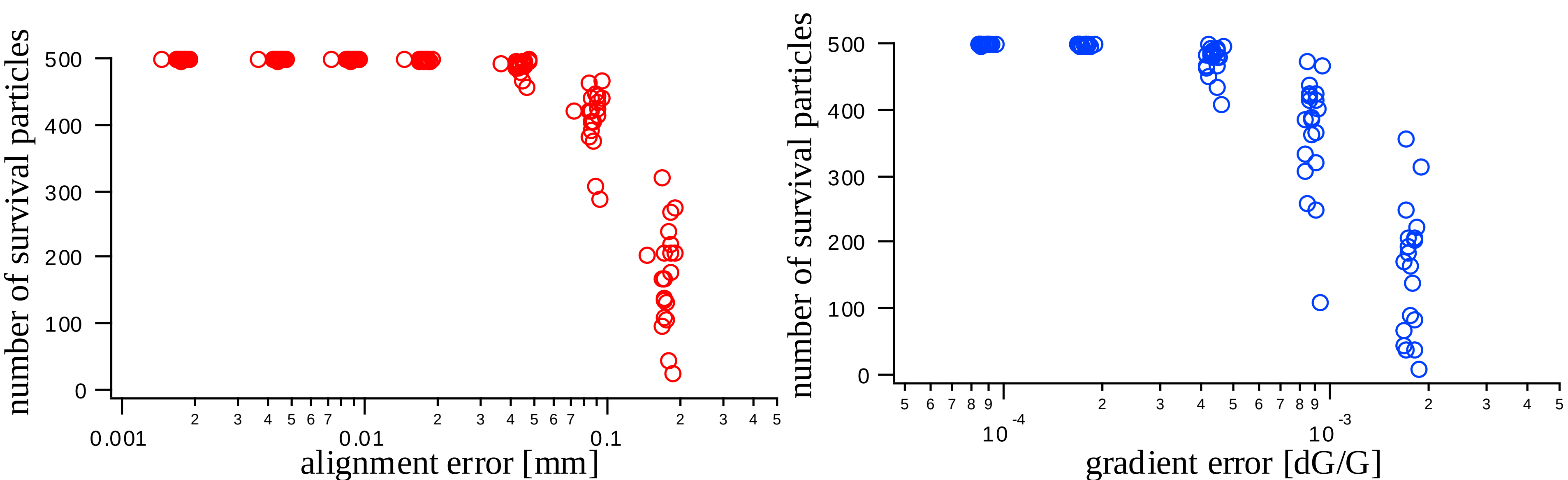
Tune Dependence on Energy



BROOKHAVEN NATIONAL LABORATORY

Error Analysis (Machida)

- Introduce magnet displacements and gradient errors
- Find that 20–50 μm displacements and 2–5 $\times 10^{-4}$ gradients are tolerable in the baseline
- Ignoring longitudinal dynamics: may complicate
- Should look at other errors: random nonlinearities, RF phase errors, others

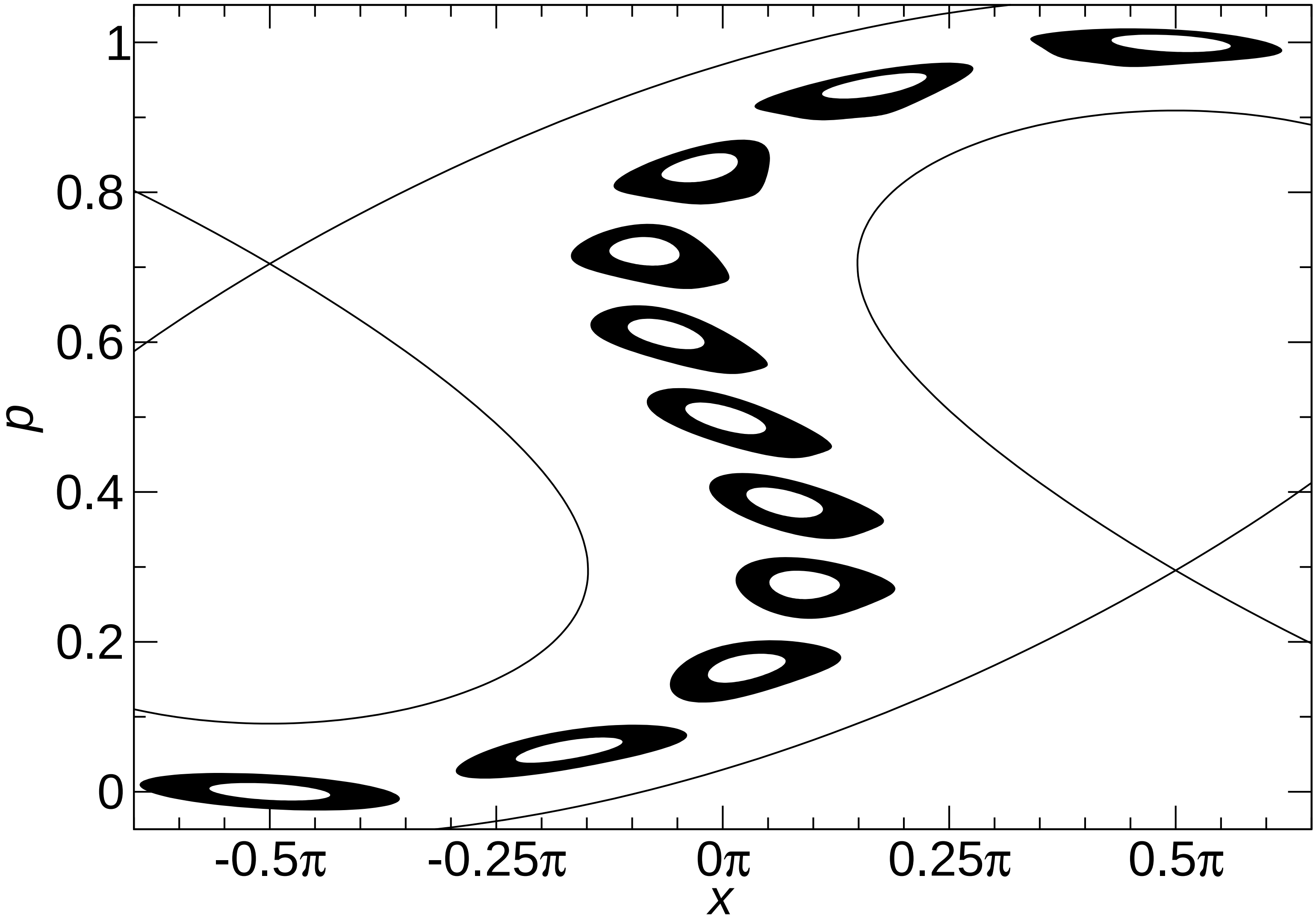


Longitudinal Dynamics

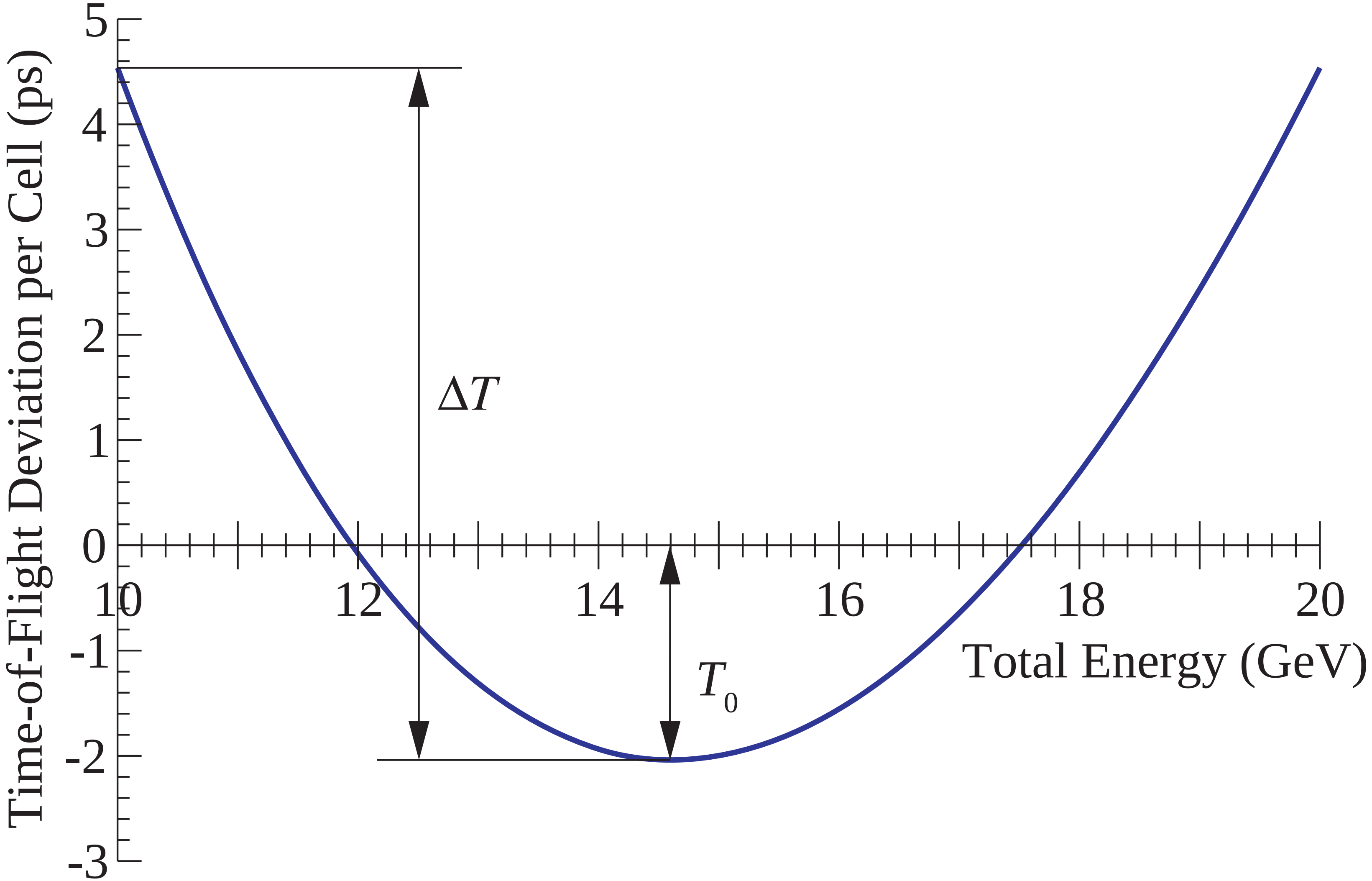
- Linear non-scaling FFAGs have unusual dynamics: particles move through channel in phase space
- Caused by time of flight dependence on energy that is isochronous at one point within energy range
- Need to understand optimal design
  - Optimal beam orientation
  - Optimal choice of machine parameters
- Studied under assumption that time of flight is symmetric parabola, and single harmonic RF
- Needs more work in more general case

BROOKHAVEN NATIONAL LABORATORY

Longitudinal Phase Space

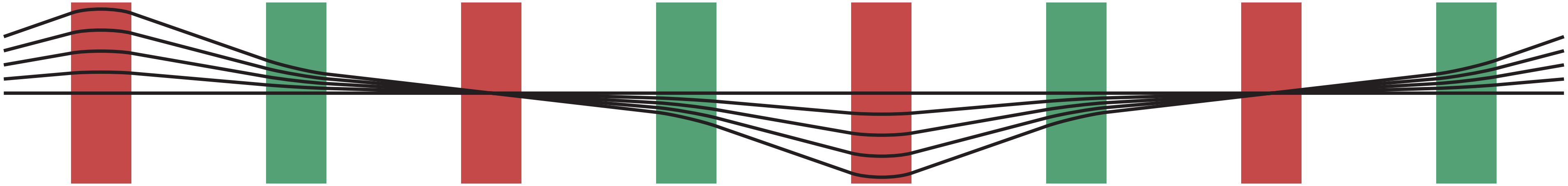


Time of Flight vs. Energy



Time of Flight Dependence on Transverse Amplitude
What is the Problem?

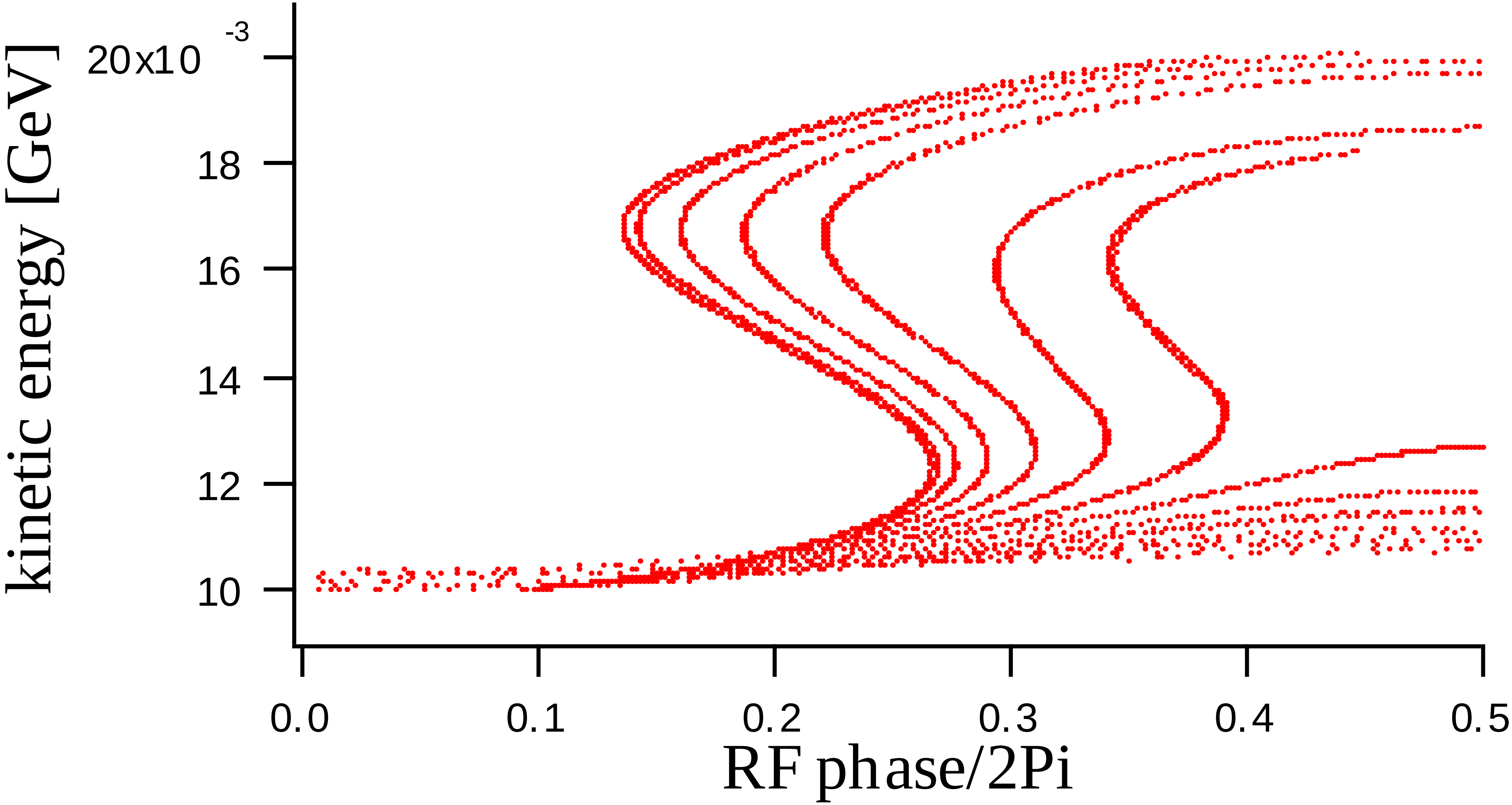
- Particles with large transverse amplitudes aren't accelerated
- Time of flight depends on transverse amplitude
- Reason: larger amplitudes, angles make longer path length



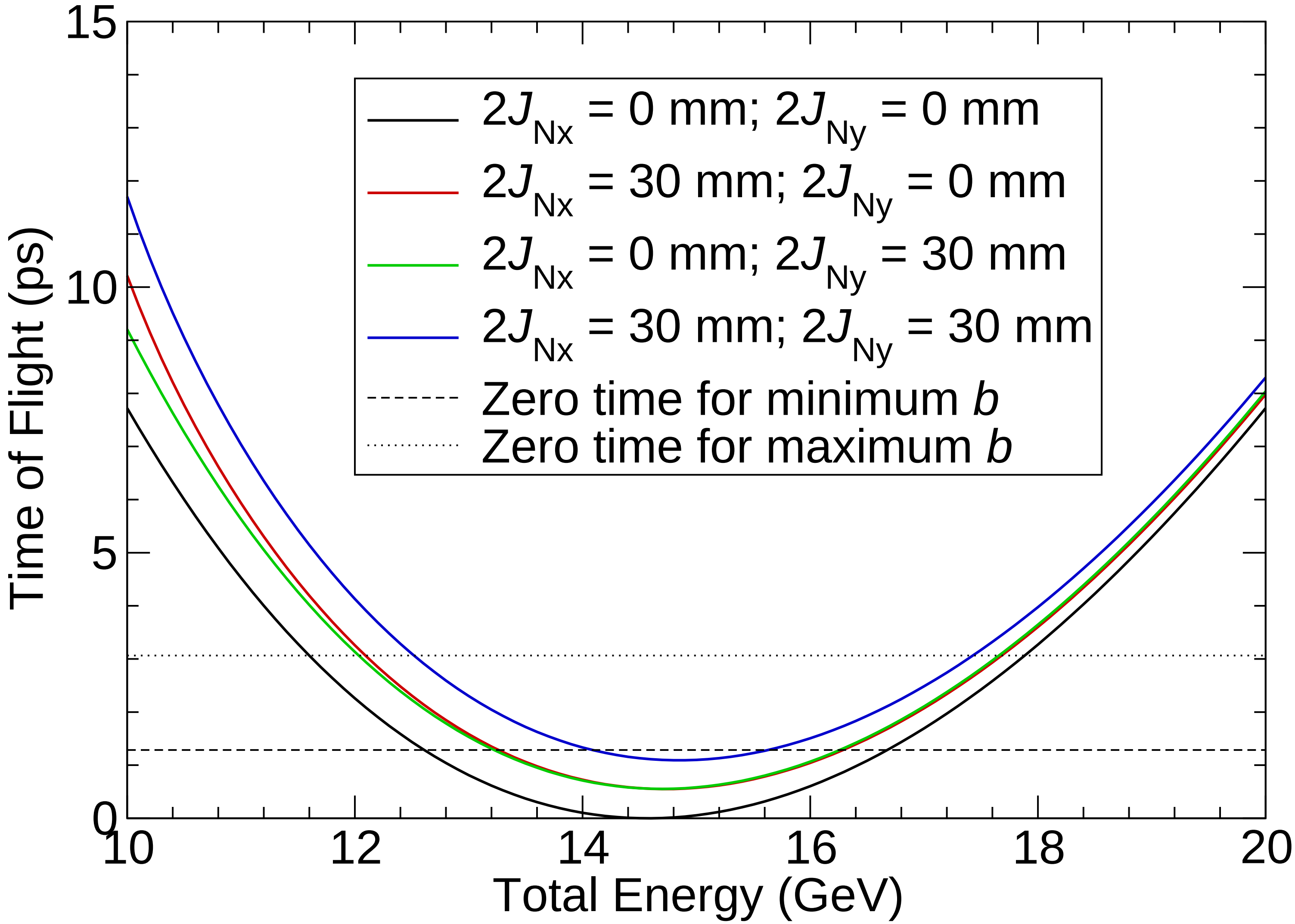
- Different times of flight for different amplitudes create acceleration problems in FFAGs
- Time of flight dependence on amplitude related to chromaticity

$$\frac{d\bar{t}}{ds} = -\partial_E H_T - \frac{2\pi(\partial_E \boldsymbol{\nu}) \cdot \boldsymbol{J}_n}{L} + O(\boldsymbol{J}_n^{3/2}).$$

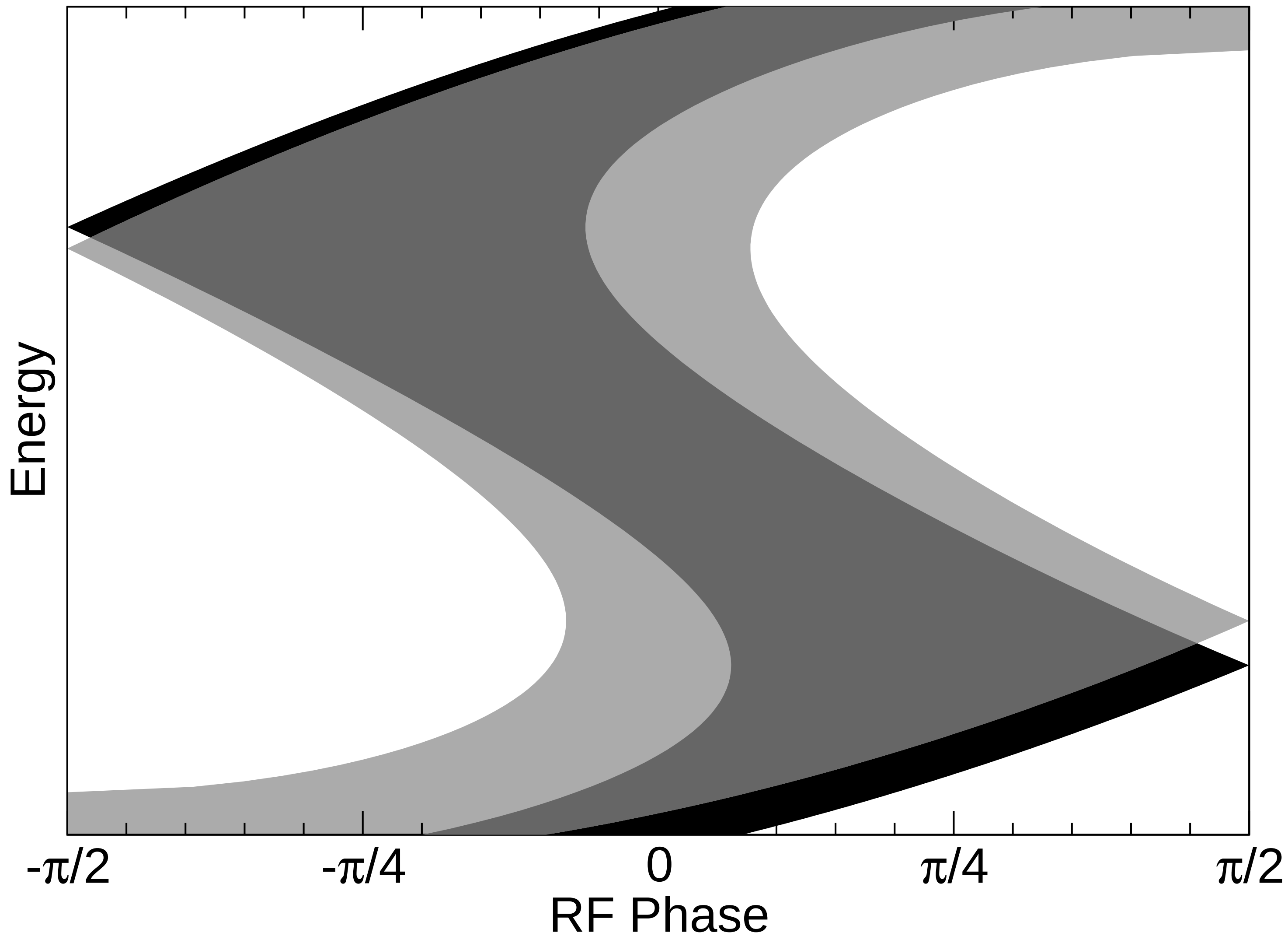
Acceleration of Particle
Different Transverse Amplitudes



Time of Flight Depends on Transverse Amplitude



Acceleration Channels in FFAGs



Plan for Addressing Time of Flight Problem

- Time of flight difference at end for uniform acceleration

  $$-2\pi\Delta\boldsymbol{\nu}\cdot\boldsymbol{J}_n/(\Delta E)$$

  $\Delta\boldsymbol{\nu}$ is tune difference from beginning to end per cell, ΔE is energy gain per cell
- Increase energy gain per cell (expensive)
- Use third harmonic RF to make phase space more forgiving (kind of expensive)
- Correct chromaticity (free!) in FFAG
- Put positive chromaticity in transfer lines

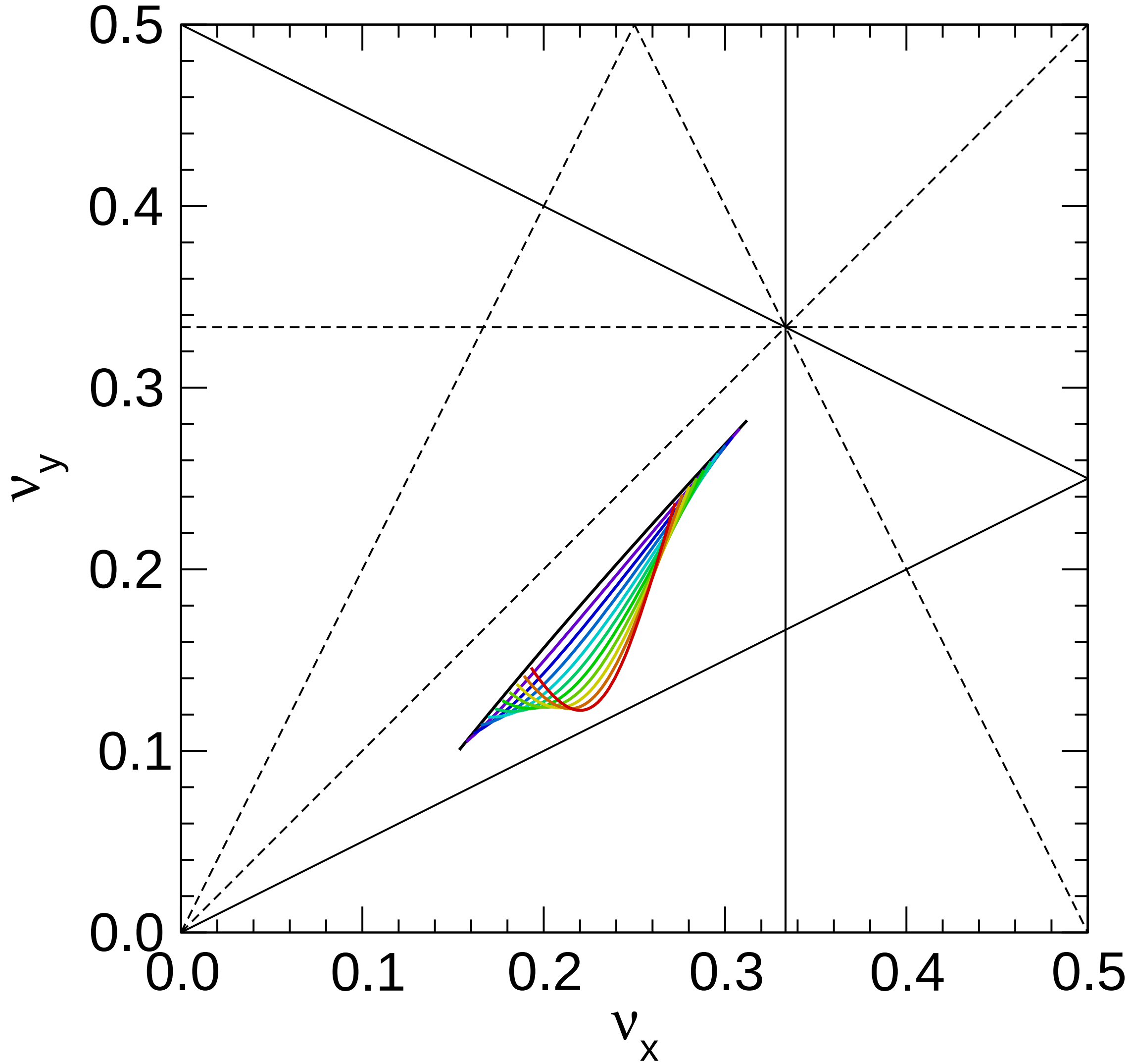
Chromaticity Correction Method

- Correct chromaticity with a sextupole component to magnets as follows
  - Construct a linear lattice where
    - Magnet lengths, drift lengths, and the number of cells are fixed
    - Time of flight is the same at low and high energy
    - The following three distances in the tune plane are equal
      - Low energy tune ($\boldsymbol{\nu}_{\text{lo},0}$) to $3\nu_x = 1$ line
      - Low energy tune to $\nu_x - \nu_y = 0$ line
      - High energy tune ($\boldsymbol{\nu}_{\text{hi},0}$) to $\nu_x - 2\nu_y = 0$ line

Chromaticity Correction Method

- Chromaticity correction procedure (cont.)
  - Add sextupole components, and modify dipole and gradient components so that
    - Magnet lengths, drift lengths, and the number of cells are fixed
    - Time of flight is the same at low and high energy
    - If x is the fraction of chromatic correction
      - $\boldsymbol{\nu}_{\text{lo}} = (1 - x/2)\boldsymbol{\nu}_{\text{lo},0} + (x/2)\boldsymbol{\nu}_{\text{hi},0}$
      - $\boldsymbol{\nu}_{\text{hi}} = (x/2)\boldsymbol{\nu}_{\text{lo},0} + (1 - x/2)\boldsymbol{\nu}_{\text{hi},0}$
- Choice of tune range to avoid third order resonances which sextupole will drive
- Plot shows to $x = 0.5$

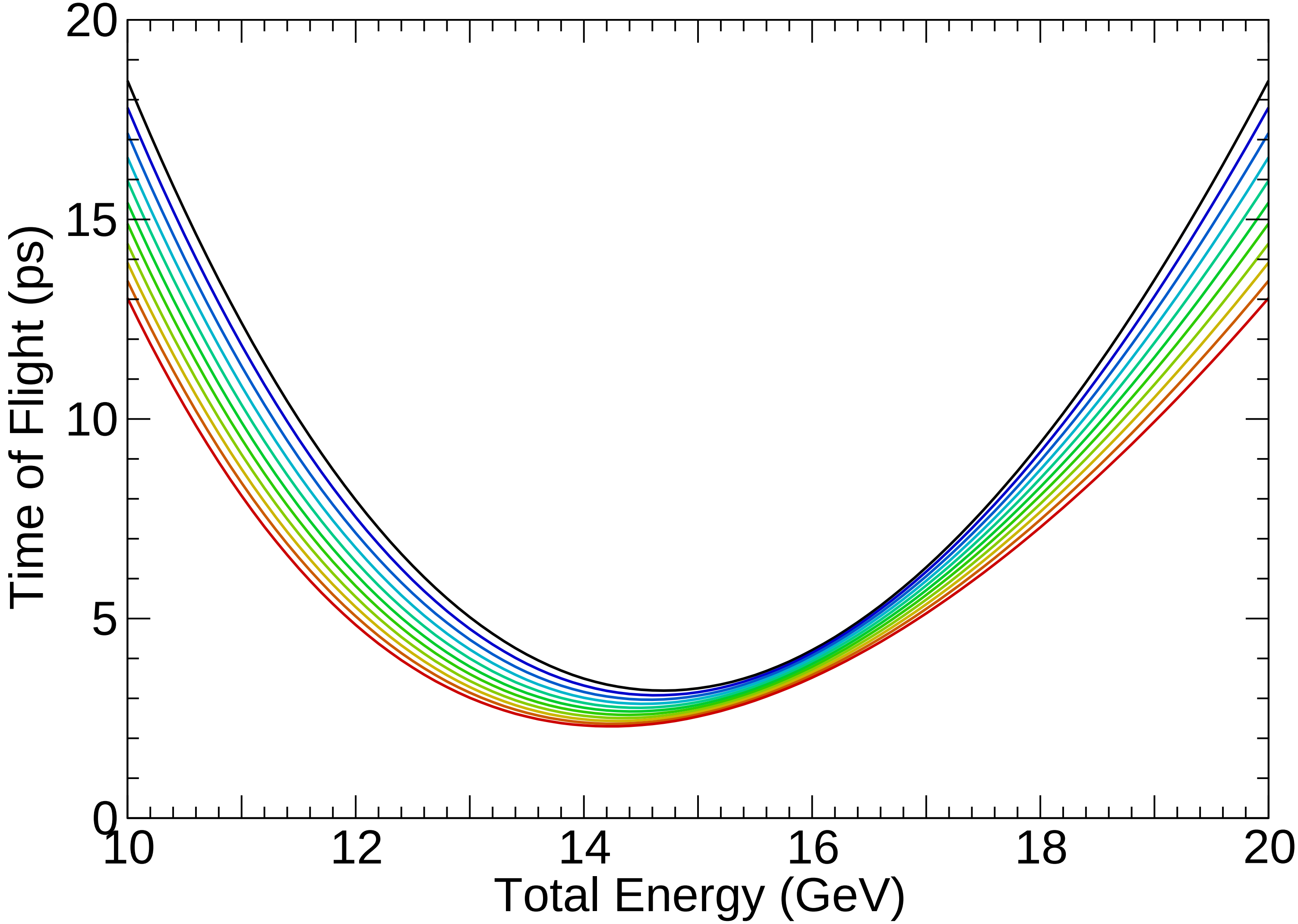
Tune Range with Chromaticity Correction



Observations

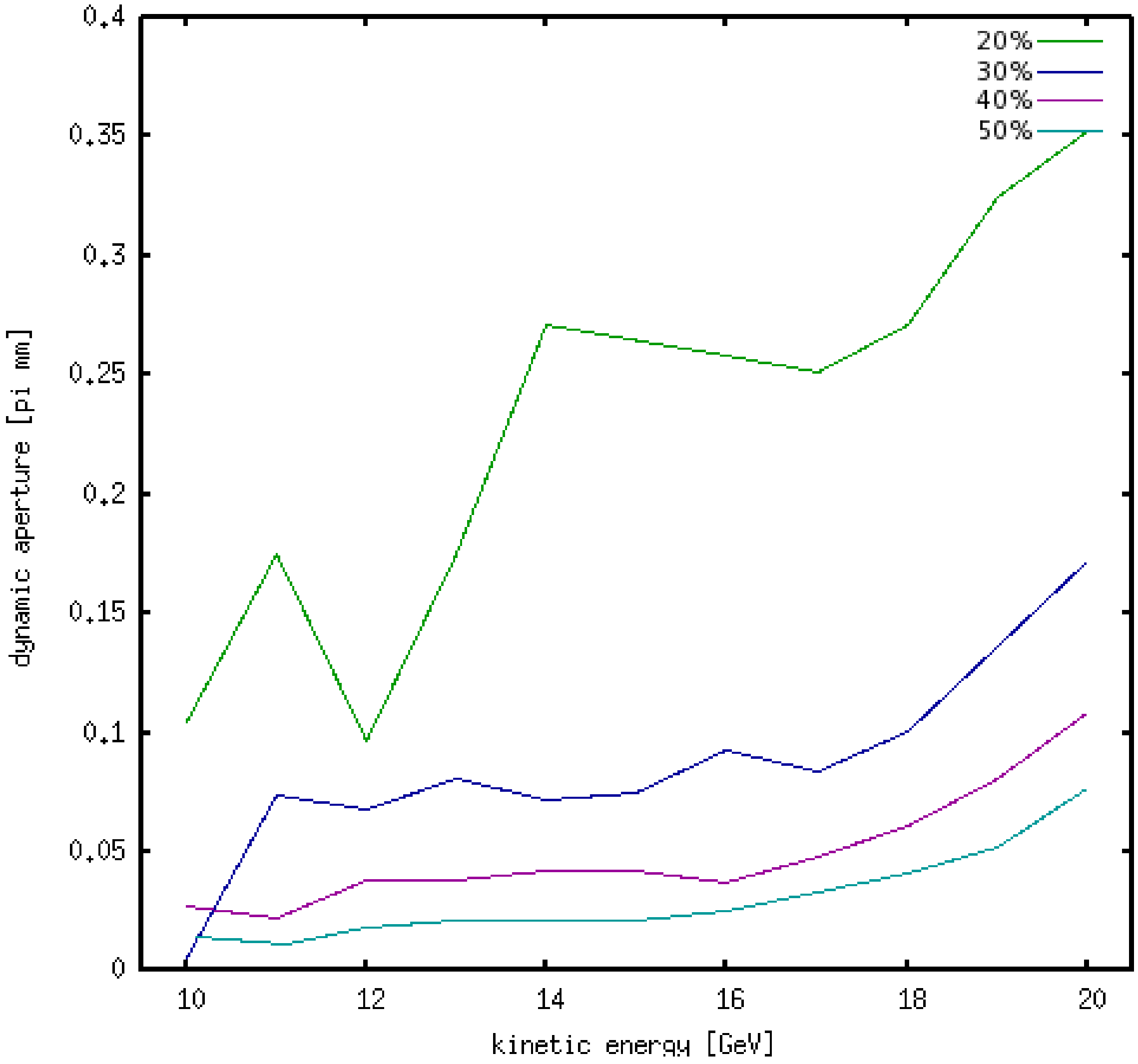
- Note chromaticity is locally higher!
- However, for uniform acceleration, what matters is the total change in tune
  - However, increased chromaticity may affect phase space locally!
- Time of flight range actually improves with more sextupole
- Must determine if dynamic aperture is sufficient
  - Losses likely on $4\nu_x = 1$ resonance
  - Should ascertain if we have decent dynamic aperture except for that

Time of Flight Variation with Chromaticity Correction



Dynamic Aperture (Machida)

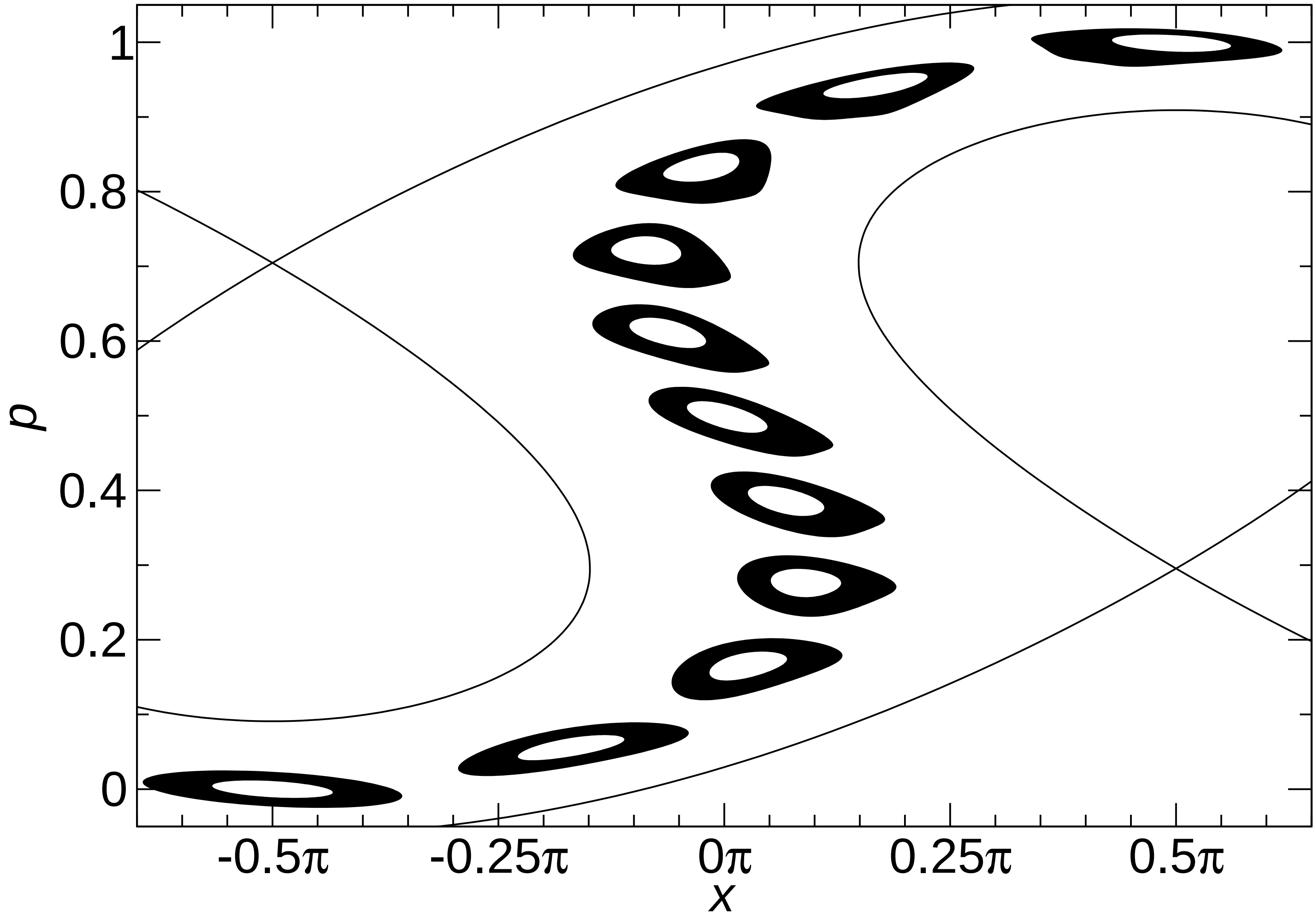
- Dynamic aperture less for higher chromaticity
- Some dynamic aperture reduction on $4\nu_x = 1$ or $4\nu_y = 1$
- 20–30% may be tolerable



Electron Model (EMMA)

- Linear non-scaling FFAG has never been built
- Would like to test whether we understand the dynamics in such a machine
- Build a 10–20 MeV model that accelerates electrons
- Test our understanding of
  - Longitudinal dynamics
  - Transverse dynamics when acelerating through many weak resonances
  - Sensitivity to errors
- In the proposal stages now, sited at Daresbury

Longitudinal Dynamics



Conclusions and Plans

- Errors can significantly degrade performance of linear non-scaling FFAGs
- We need to understand the unusual longitudinal dynamics of these machines to make optimal use of them
- Time of flight dependence on transverse amplitude is a significant difficulty which must be addressed
  - We have a plan of attack
- We are hoping to build an electron model to test our understanding of linear non-scaling FFAGs